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PROBLEM OF CONSTRUCTING A STEP FUNCTION FLUCTUATING LEAST AROUND A GIVEN FUNCTION

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Abstract. It is well known that step functions are often used in regulation and optimal control. Nevertheless, to our knowledge, this problem has not been raised previously. Therefore, the problem of constructing a step function, least fluctuating around a given function (in the sense of integration), is the main point of this article. This is a multiextremal task. In particular, such problems mostly arise in the design.

In this paper formulated a mathematical model of the problem. To solve the problem developed an effective method of shrinking neighborhoods. Moreover, for algorithm of the method was developed software.

Keywords: step functions, calculus of variations, optimal control, pipelines, pumping stations, regulating tanks.

AMS Subject Classification: MSC2010 49K35, 65K10.

1. Introduction

A function with a value remaining constant at each of a series of finite intervals is called step function. It is known that step functions are used in modeling of various events and processes. It is a function which is frequently used especially management systems. For this reason, the problem of constructing a step function in the sense of integration least fluctuating around a given function arises. This problem is a problem arising in mathematical modeling of Water Supply Systems (WSS). It is especially observed in the problem of creating a graphic indicating the working of pump station for 24 hours so that the volumes of water tanks will fall to minimum level. Related to this problem, the problem of construction of the step function was put forth for the first time by Bayraktarov B.R. and Kudayev V. Ch. [4].

WSS as a complex system is a system composed of pipelines (water supply network); pressure and regulation plants (pump station and water tanks). The highest cost part of the system is the pipelines. For this reason, many researchers [2, 3, 6, 9, 10, 13, 14, 15, 19, 20] have examined the problem of calculating the most appropriate parameters of water supply networks (hydraulic parameters of pressure and pipes at the top of the network). These parameters are the values composing the parameters of the plants of the system. Many effective methods have been proposed related to the solution of this problem. For example, linear programming (LP) [2, 9], dynamic programming and gradient method in networks with many sources [5, 20] and various genetic and hybrid algorithms [6, 10, 14, 15].

To minimize operating costs, there are studies related to the planning of working of pump stations and the determining of the volume of the tank. Of these, the method combined based on the gradient and punishment methods (U. Shamir [19]), the heuristic method (Ormsbee, L.

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E., Lansey, K. E. [13]), dynamic programming (Lansey, K. E., Awumah, K. [11]), non-linear programming (Ilemobade, A. A., Manson, N. J., Stephenson, D. [7]) and different methods based on various genetic algorithms (Savic, D. A., Walters, G. A., Schwab, M. [17], Boulos, P. F., Wu, Z., Orr, C. H., Moore, M., Hsiung, P. [5], Thomas, D., Lopez-Ibanez, M., Prasad, T. D., Paechter, B. [12], Seleka, ., Benea, J. G., Hosb, C., [18]) can be cited as examples. Besides these, Sarbu, I. and Kalmar, F. ([16]) also suggested that the pump station should work completely automatically for not using the tanks. The researchers in their study ([5, 12, 18]) made a more comprehensive analysis of current methods and algorithms related to the problem.

As Keedwell, E. and H. Soon-Thiam ([10]) state, in recent years genetic algorithms have become one of modern methods in the organization of technical systems and especially WSS's. The initial solution of genetic algorithms is generally random. This selection "might not be good" and later might not produce an effective solution at a required level. Building a "good" initial solution for genetic algorithms preserves the place of the problem on the agenda.

WSS a very complex system and the effectiveness of this system depends on many factors. Naturally it is not possible to include all of these factors in a mathematical model. As stated above, the highest cost part of the system is the water supply network. For this reason, what is the most effective in the system are the parameters of the network (amount of pressure at the top of the network and hydraulic parameters of the branches). Another part of the system – pressure and regulating plants (pump station, water tanks) are the parts which are in interaction with only one another. The effective parameters of this part are pressure amount (height of the tank lever) and the regulating volume of the water tank depending on the working graphic of the pump station. For this reason, the mathematical modeling of WSS is composed of two problems: one of these – convex programming problem (calculation of the optimal parameters of water supply network), and the other one is variance calculation problem (creating the graphic indicating the working of the pump station making the regulating volume of the water tank minimum).

This study shows that the considered variation problem (construction of a step function) is multiextremal and, at the same time, is a minimax problem. It was also shown that the local minimum solution found has global minimum. An effective method related to the solution of the problem was suggested:

- Since the problem is multiextremal, not one but K number of most appropriate initial (rough) solutions can be built;
- By applying the method of "narrowing intervals" to each initial solution, the most appropriate solution was produced.

The results obtained as a result of the numerical experiment made showed that the suggested method is highly effective in terms of both the value of the objective function and the duration of calculation time [4].

It was concluded that the method suggested for the solution of the construction problem of the step function in mathematic modeling of complex systems including WSSs as a heuristic method can be used in building "good" initial solutions in various genetic algorithms.

2. Mathematical model of the problem

Without going into technical details, we consider the problem of constructing a step function $x(t)$ on an interval $[a, b]$ having at most n steps, in the integral sense of fluctuating about a given function $g(t)$:

$$\max_{a\le\alpha\le b}\int_a^{\alpha}[g(t)-x(t)]\,dt-\min_{a\le\beta\le b}\int_a^{\beta}[g(t)-x(t)]\,dt\to\min \tag{1}$$

$$\int_a^b g(t)dt=\int_a^b x(t)dt. \tag{2}$$

In the task values of α,β and $c_1,c_2,\ldots,c_n$,$t_1,t_2,\ldots,t_{n-1}$, parameters of the step function should be determined

$$x(t)=\begin{cases}c_1, a\le t<t_1,\\ c_1, t_1\le t<t_2,\\ \vdots\\ c_n, t_{n-1}\le t<b.\end{cases}$$

The defined problem is a minimax problem defined

$$\max_{a\le\alpha\le b}\int_a^{\alpha}[g(t)-x(t)]\,dt-\min_{a\le\beta\le b}\int_a^{\beta}[g(t)-x(t)]\,dt=\max_{a\le\alpha\le b}\int_a^{\alpha}[g(t)-x(t)]\,dt+$$

$$+\max_{a\le\beta\le b}\int_{\beta}^{a}[x(t)-g(t)]\,dt\Rightarrow$$

$$\Rightarrow\max_{\substack{a\le\alpha\le b\\ a\le\beta\le b}}\left[\int_a^{\alpha}|g(t)-x(t)|\,dt+\int_{\beta}^{a}|g(t)-x(t)|\,dt\right]=\max_{\substack{a\le\alpha\le b\\ a\le\beta\le b}}\int_{\beta}^{\alpha}|g(x)-x(t)|\,dt$$

as

$$\max_{\substack{a\le\alpha\le b\\ a\le\beta\le b}}\int_{\beta}^{\alpha}|g(x)-x(t)|\,dt\ \max_{\substack{a\le\alpha\le b\\ a\le\beta\le b}}\left|\int_{\beta}^{\alpha}[g(x)-x(t)]\,dt\right|\Rightarrow\max_{\substack{a\le\alpha\le b\\ a\le\beta\le b}}\left|\int_{\beta}^{\alpha}[g(x)-x(t)]\,dt\right|\to\min \tag{3}$$

with the observance of a ratio of the balance (2).

As far as we know, the task has not been previously put forward, though regulation and optimum control step functions have often been used.

3. Problem of determining optimal regulating volume of water in WSS's

In pipeline systems (water, oil, gas pipelines, etc.), an obligatory element is the tanks used for storage, clearing and further transportation of substance and pump stations. Tanks and water towers as regulating capacities everywhere are used as systems of water supply of settlements. Therefore, definition of the regulating volume of tanks is a practical problem. Moreover, definition of the minimum volume will allow reducing expenses for building of these constructions.

The regulating capacity of tanks depends only on the hourly schedule of consumption and from the schedule of work of pump stations of system. In practice, this volume is defined by combination of these two schedules.

At each hour every day, a certain percent from consumption total for days and this quantity in intervals $[t_i,t_{i+1}]$, i $=1,2,\ldots,23$ is consumed which is considered conditionally constant, i.e. the schedule has a step appearance (see Figure 1).

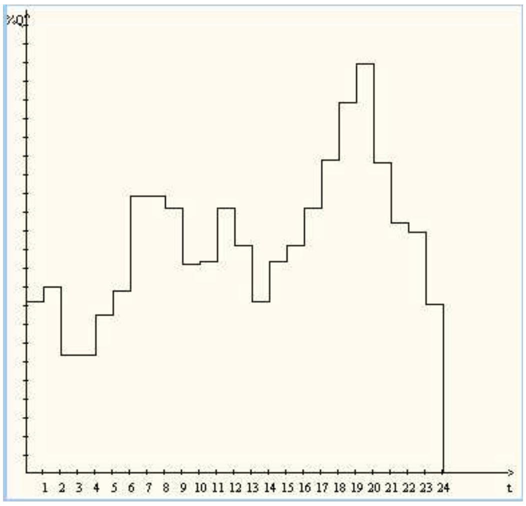

Figure 1. The hourly schedule of consumption.

Unlike the consumption schedule (about 24 steps) on specifications on operation, the schedule of work of pump station can have no more than three – four steps. A case when the schedule of consumption within days uniform (or it is close to it) does not represent interest (in this case the regulating capacity it is equal to zero). In other cases, it is possible to define the schedule of work of pump station so that the regulating volume will be minimum and fulfill necessary requirements.

In Figure 2, both schedules (a red line – the schedule of work of pump station) are presented. On this schedule, light painted area is not sufficiently giving pump station and dark painted area is a superfluous giving.

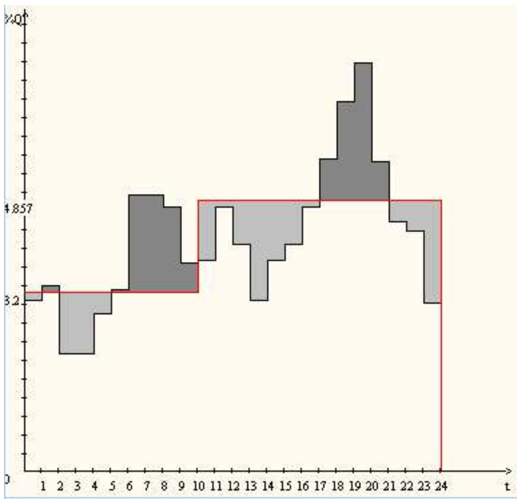

Figure 2. The combined schedules.

The working principle of the system consists of the followings: water at pump station insufficiently given to the pipeline, deficiency of substance moving in system from the tank (Fig.3a), and surplus giving from pump station – surplus arrives in the tank (Fig. 3b).

Necessary condition of definition of the schedule of work of pump station – the algebraic sum of the areas of the painted areas should be equal to zero. Within day's water which arrives in the tank follows from it, naturally, during any moment of time in the tank, the greatest quantity of water will collect. It is obvious that values of the regulating volume of the tank are the greatest value of current water – supply in the tank [1].

For the decision of this problem in Karambirov S. N. ([8]) as one of ways of reception of the schedule of characteristic modes, it is offered to sort the schedule of water consumption with its subsequent averaging on several characteristic intervals. For definition of intervals of averaging, the scheme

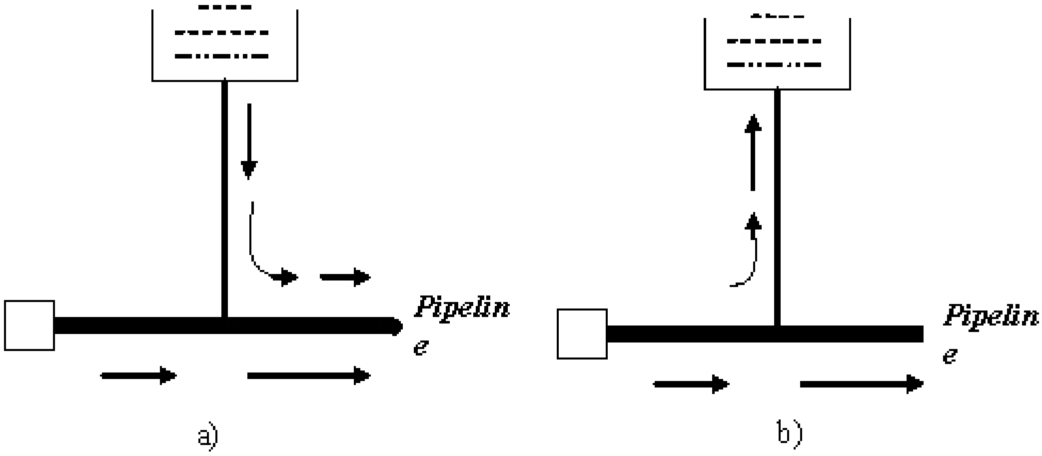

Figure 3. The working principle of the system.

of dynamic programming is used. The basic disadvantage of the offered method, in my opinion, is the averaging of the schedule of consumption on the allocated characteristic intervals.

4. The problem formulation

At substantial level, the problem consists of the followings: at the known daily hourly schedule of consumption to define the schedule of work of pump station so that the regulating volume of tanks was minimum.

Let

n – Quantity of steps of work of pump station;

$q = \{q_i\}$–The set sizes of expenses of water consumed by system at each o'clock

(a set in percentage of the total of water consumed within days);

$x = \left\{x^{(j)}\right\}$–Required sizes of expenses of water (in percentage), submitted pump station in system at each o'clock j the period of its work;

$t = \{t_j\}$–Duration of steps of work of pump station;

where, $i = 1, 2, \ldots, 24, \quad j = 1, 2, \ldots, n.$

As it has been stated above, the regulating volume of the tank is the greatest current water – supply which is available in the tank during any moment of time of days. Hence, from all possible variants of the schedule of work of pump station, we should find the schedule of work having the minimum greatest value of a current water – supply in the tank. Thus, we have a problem on a minimum from maxima.

For convenience of record of a problem, we will enter an additional designation

$$x_i = x^{(j)}, \quad \forall i \in (\sum_{m=0}^{j-1} t_m, \sum_{m=1}^{j} t_m], \quad t_0 = 0, \quad j = 1, 2, \ldots, n, \quad i = 1, 2, \ldots, 24 \tag{4}$$

then the regulating volume of the tank will be

$$W(q,x,t) = \max\sum_{i=0}^{p_1}(q_i - x_i) - \min\sum_{i=0}^{p_2}(q_i - x_i) \to \min, \quad p_1, p_2 = 1, 2, \dots, 24, \tag{5}$$

where t time moment on which the water – supply in the tank is defined.

Obviously, the first restriction is the sum of durations of all steps of work of pump station 24 hours should be equal, i.e.

$$\sum_{j=1}^{n} t_j = 24. \tag{6}$$

The second restriction follows from a necessary condition of definition of the regulating volume – the algebraic sum of the areas of the top and bottom shaded areas should be equal to zero and if t_j^* - the moments of transition of the schedule on other step of work

$$\sum_{j=1}^{n}\left(\sum_{i=t_{j-1}^*+1}^{t_j^*}(q_i - x_j)\right) = 0,$$

where

$$t_j^* = \sum_{j=1}^{j} t_j, \quad j = 1, 2, \dots, n, \quad t_0^* = 0$$

opening this sum, we obtain the following equation

$$\sum_{i=t_0^*+1}^{t_1^*}(q_i - x_1) + \sum_{i=t_1^*+1}^{t_2^*}(q_i - x_2) + \dots + \sum_{i=t_{n-1}^*+1}^{t_n^*}(q_i - x_n) = 0$$

or

$$\sum_{i=t_0^*+1}^{24} q_i - \sum_{j=1}^{n}(t_j^* - t_{j-1}^*)x_j = 0,$$

Let's definitively receive the second restriction

$$\sum_{\imath=1}^{24} q_\imath = \sum_{j=1}^{n} t_j x_j, \;\; where \;\; t_j = t_j^* - t_{j-1}^*. \tag{7}$$

Taking into account (5) - (7) and also considering that $x_j \in [q_{\min}, q_{\max}]$ mathematical model of a problem, it is possible to present the values of required variables as a kind:

$$W(q,x,t) = \max\sum_{i=0}^{p_1}(q_i - x_i) - \min\sum_{i=0}^{p_2}(q_i - x_i) \to \min, \quad p_1, p_2 = 1, 2, \dots, 24 \tag{8}$$

$$\begin{cases} \sum\limits_{j=1}^{n} t_j x_j = \sum\limits_{\imath=1}^{24} q_\imath, \\ \sum\limits_{j=1}^{n} t_j = 24, \\ q_{\min} < x_j \leq q_{\max}, \end{cases} \tag{9}$$

where $q_{\max} = \max\limits_{t}\{q_t\}$, $t = 1, 2, \dots, 24, \quad j = 1, 2, \dots, n-1.$

5. Method and algorithm scheme of problem solution

It is obvious that the bigger the number of steps, the less the variation of the required function. However, it is also likely that any increase in the number of steps significantly increases the discounted cost of the entire real-world system. Therefore, the acceptable maximum number of the steps of the required functions is limited. For instance, this number for WSSs (pump stations – regulating tanks) is limited by four steps.

As noted in the article by Yelsakov S.M. and Shyryaev V.I. [21], any impossibility to use an analytical approach for solving the problems of multiextremal optimization results from either complicated representation or algorithmic setting up of objective function and/or limitations. In both cases, the objective function is considered as a ‘black box’ and the main factor of solving the problem is the time of calculation, i.e. the best solution is obtained within a reasonable time length. This factor of “reasonable time length” can be different for different real-world systems.

As it is seen from the statement of the problem, it is a minimax problem. We will testify that the problem has an absolute minimum for lengthwise fixed series of steps of the required function.

Really, let the representation (3) be used and show that this problem comes down to the linear programming problem.

Let an arbitrary decomposition of interval [0; 24] into n parts (steps) be given by points $\tau_1, \tau_2, \ldots, \tau_{n-1}$ and k, l be two arbitrary points out of [0; 24], where $k \in [\tau_{m-1}, \tau_m]$ and $l \in [\tau_{p-1}, \tau_p]$.
Then the problem in the lengthwise fixed series of steps is as follows:

$$W(c_1, c_2, \ldots, c_n) \to \min . \tag{10}$$

A consequence of properties of the module (3) is

$$\sum_{\imath=k}^{\tau_m} g_i - c_m(\tau_m - k) + \sum_{\imath=\tau_m}^{\tau_{m+1}} g_i - c_{m+1}(\tau_{m+1} - \tau_m) + \ldots + \sum_{\imath=p-1}^{l} g_i - c_p(l - \tau_{p-1}) \le W, \tag{11}$$

$$-\left(\sum_{\imath=k}^{\tau_m} g_i - c_m(\tau_m - k) + \sum_{\imath=\tau_m}^{\tau_{m+1}} g_i - c_{m+1}(\tau_{m+1} - \tau_m) + \ldots + \sum_{\imath=p-1}^{l} g_i - c_p(l - \tau_{p-1})\right) \le W, \tag{12}$$

$$c_1\tau_1 + c_2(\tau_2 - \tau_1) + \ldots + c_2(b - \tau_{n-1}) = \int_0^{24} g(t)dt, \tag{13}$$

$k = 1, 2, \ldots, n;\;\; l = 1, 2, \ldots, n-1; k \ne l$, where - the sought step values (flow rates), W – the sought optimal functional value.

The system of inequities (11), (12) contains $2n(n-1)$ linear inequities, i.e. this is a linear programming problem with respect to the unknown $\mathbf{c}$. Consequently, at the lengthwise fixed series of steps the local minimum of the problem is then an absolute minimum; at the same time this means that the problem is multiextremal.

Because the problem is not only minimax but also multiextremal, the objective function is defined algorithmically; therefore, any analytical methods of optimization are not suitable; in this context the problem is considered as a combinatorial one, namely the problem on permutations (exhaustive). Unfortunately, no such properties of extremum have been discovered to enable us to reduce significantly the number of fingering when solving the problem.

It is known that the problem of the calculus of variations often decide first on a coarse grid, and then in the vicinity of the solution to decide on a fine grid. Such an approach makes it possible to reduce significantly the search time for the best solution. The suggested method represents the modification of this method and, different from this method, it involves the consequent narrowing of the neighborhoods obtained at the latest iteration of the problem solution. The narrowing of the neighborhood and solving the problem therein is made with the increasing accuracy until a solution with the given accuracy is obtained. Herewith the solution is sought by a directed exhaustive search (in each neighborhood) each time. The number of exhaustions of the solutions in each of ever-more narrowing neighborhoods remains approximately the same (selectable initially by trial method when selecting the step in the first (rough) grid. Each next neighborhood involves one step of previous iteration. Moreover, in view of that fact that the problem is multiextremal, the descent is made simultaneously from several initial solutions.

For each of initial solutions, any iteration is stopped when on two consecutive runs within the whole interval of time [0; 24] the value of object function will differ by not more than given accuracy of the problem solution.

Solving the problem (general algorithm of solution is represented in Fig. 4.) is made in three stages.

At the first stage, the K number of the best rough solutions is built on full set of valid values of water discharge. For this purpose, the search interval [$\mathrm{q}_{\min}$, $q_{\max}$] is consecutively decomposed into the N parts for each of them with step $(\Delta x)^{(k)} = \dfrac{q_{\max} - q_{\min}}{N}$ (here $k = 1, 2, 3, \ldots, K, \quad N = k + 3$) all kinds of couples, $(t, \ x)$ meeting the conditions of the problem are generated and the best rough value of the sought function is determined out of them.

At the second stage, an iteration process is built for each of the K solutions at each step which search area of water flow rate values for each sought period of $[t_p^{(k)}, \ t_{p+1}^{(k)}]$ is to be given separately by intervals, $[x_p^{(k)} - (\Delta x)^{(k)}, \ x_p^{(k)} + (\Delta x^{(k)})]$ in which, at step $\dfrac{2(\Delta x)^{(k)}}{N}$, all kinds of couples (t, x) are generated and the best solution is calculated. This process runs on until the given accuracy of the required solution is achieved at two consecutive iterations.

At the third stage, the minimal one that is the best solution of the problem is selected among the obtained K solutions.

At each stage, all kinds of possible pairs $(t, \ x)$ meeting the conditions of the problem are generated by the directed exhaustion.

Let at the i-th step of solving the problem $t^{(i)} = \{t_{i1}, t_{i2}, \ldots, t_{in-1}\}$ −vector of free variable values of time (length of the periods), and, corresponding to this vector, the vector of free variable values of flow rates $x^{(i)} = \left\{x_{i1}, x_{i2}, \ldots, x_{in-1}\right\}$ which components are determined by the relation $x_{ir} = m_{ir}\Delta x$, where $r = 1, 2, \ldots, n-1$, and m_{ir} - natural numbers, Δx - step of change in x, and the step in time for this problem is fixed and equal to one.

Then, the following can be expressed from the limitations (6)

$$x_{in} = \frac{\sum_{j=1}^{24} q_j - t^{(i)} \cdot x^{(i)}}{t_{i4}}, \quad and \quad t_{in} = 24 - \sum_{j=1}^{n-1} t_{ij},$$

for various natural numbers $\left\{m_{\imath 1}, m_{\imath 2}, \ldots, m_{\imath n-1}\right\}$ with repetition (as the equation of the values of sought variables x is assumed), each of which belongs to interval $\left[1, \quad \left\|\frac{q_{\max}}{\Delta x}\right\|\right]$ and at which $x_{in} \leq q_{\max}$. Any repositioning of these natural numbers is allowed. It is not difficult to determine

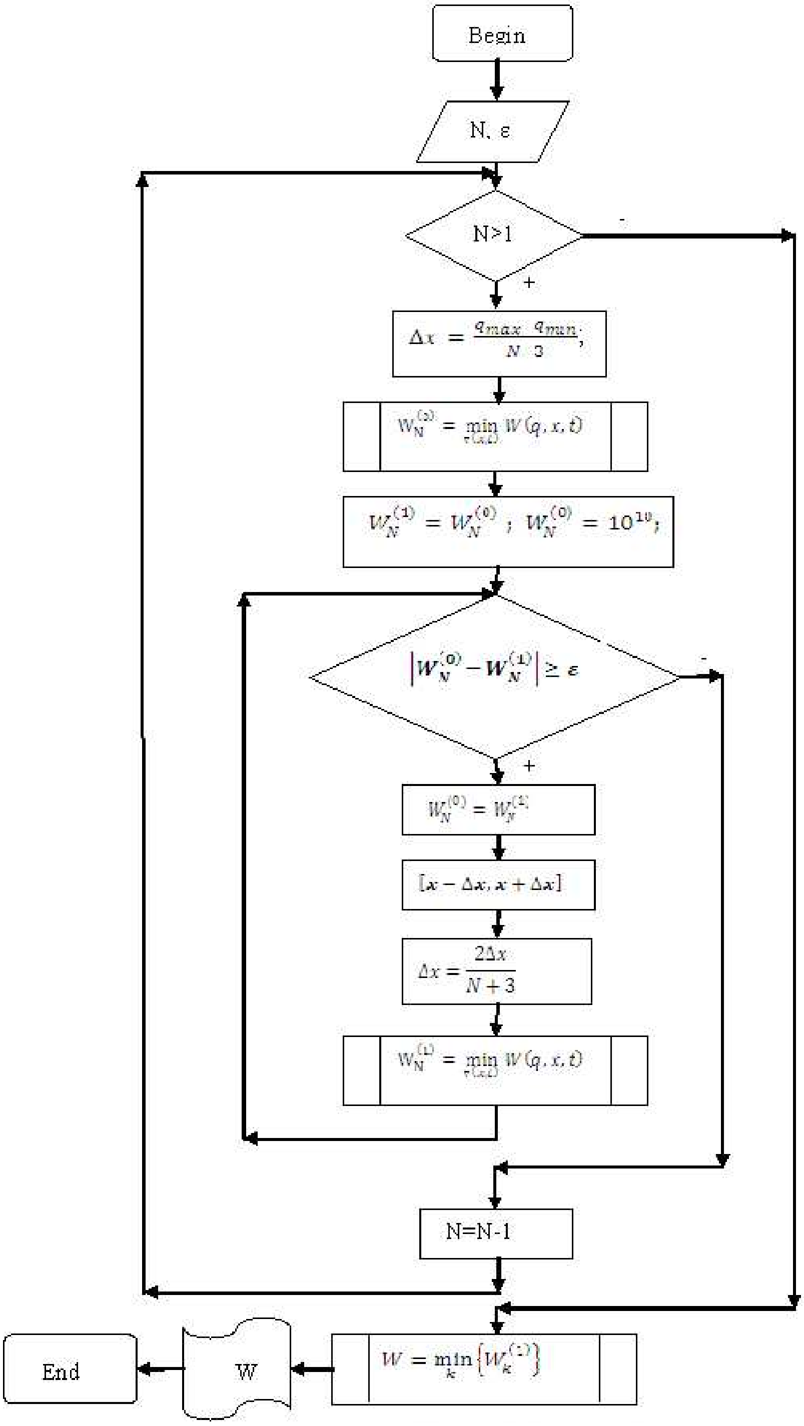

Figure 4. General scheme of algorithm.

that the maximum number of possible variations will be approximately equal

$$(n-1)!\tilde{C}_{24}^{n-1} \cdot (n-1)! \cdot \tilde{C}_s^{n-1} = \frac{(22+n)!}{23!} \cdot \frac{(s+n-2)!}{(s-1)!},$$

where $s = \left\| \frac{q_{\max}}{\Delta x} \right\|$, and $\tilde{C}_s^{n-1}$ number of combinations with repetitions and, as it was said before, $n << 24$.

For fixed values of water consumption $(x^{(i)})$and the duration period $(t^{(i)})$is calculated for each hour the amount of water in the tank [21]. To calculate the volume of water in the regulatory capacity at any point in time as the reference point should be taken that point of time p, where the value of $\sum_{j=1}^{p}(q_j - x_j)$ has the lowest value.

By assuming that at the moment of time $t^{'}$ the least value (14) is achieved, then in order to have the water stock at any other moment of time $t$the sequence is constructed $\{w_t\}$:

$$w_t = w_{t-1} + (x_t - q_t),$$

here

$$w_0 = w_{24}, t = \begin{cases} l, & if \quad l \leq 24, \\ l - 24, & if \quad l > 24, \end{cases} \quad and \ l = t^{'} + 1, \ \ t^{'} + 2, \ldots, t^{'} + 23.$$

The value of regulating volume at the i-th iteration is calculated from the relation

$$W_i = \max_{1 \leq t \leq 24} \{w_t\} - \min_{1 \leq t \leq 24} \{w_t\}.$$

6. Numerical experiments

The conducted numerical experiments showed high efficiency of the problem algorithm from the point of view of both time required for calculation and the obtained best value of functional. The calculation results are represented for one of examples (Fig. 5).

The calculation file *f01.txt* that contains the values of consumption as percent of the total water flow rate, accuracy of solution ($\varepsilon = 0.001$) and number of initial solutions ($K= 9$) is selected on Input Data bar.

“Table of Target (Output Data)” bar represents both the basic data (columns “Time” – time period and “Consumption” – consumption value in percents), and the calculation results (columns “Delivery” – delivery value, “Stock” – integral variation of function and “Stock +” – accumulation of water in the tank).

The bars “Rough solution” and “Thin the solution” represent in sequence the current rough value and thin solution of the problem for each value of N.

The bar “plot” represents three plots corresponding to the columns “Consumption”, “Delivery”, and “Stock”.

The bar “Output Data” displays the optimal solution and total consumption in cubic meters.

The maximum and minimum values of functional are displayed in the column “Stock”, and in column “Stock +” – an optimal solution (W =1.114%) of the problem; the tanks appeared to be empty (stock is equal to zero 0) in the same column within the period of 16 – 17 pm.

The bottom part of the form represents the number of conducted iterations 37, and the time of calculation 41 sec.

For this example (Fig. 6.), 14 initial solutions are constructed by decomposition of the search interval into N = 5, 6,. . . ,18 parts and for each of them the best solutions are obtained. For the illustrative purposes in Figure 6, a plot is represented showing the changes in value of the best solution depending on the number of decompositions of the search interval. As is clear from the results of the solution, the best value of functional at *N=10* (value of functional W = 1.1135662) and at *N=17* (value W = 1.113544) differ by less than 10-4.

The screen forms (Fig. 7, 8) below represent the results of solving the problem for test cases for which an absolute minimum was found.

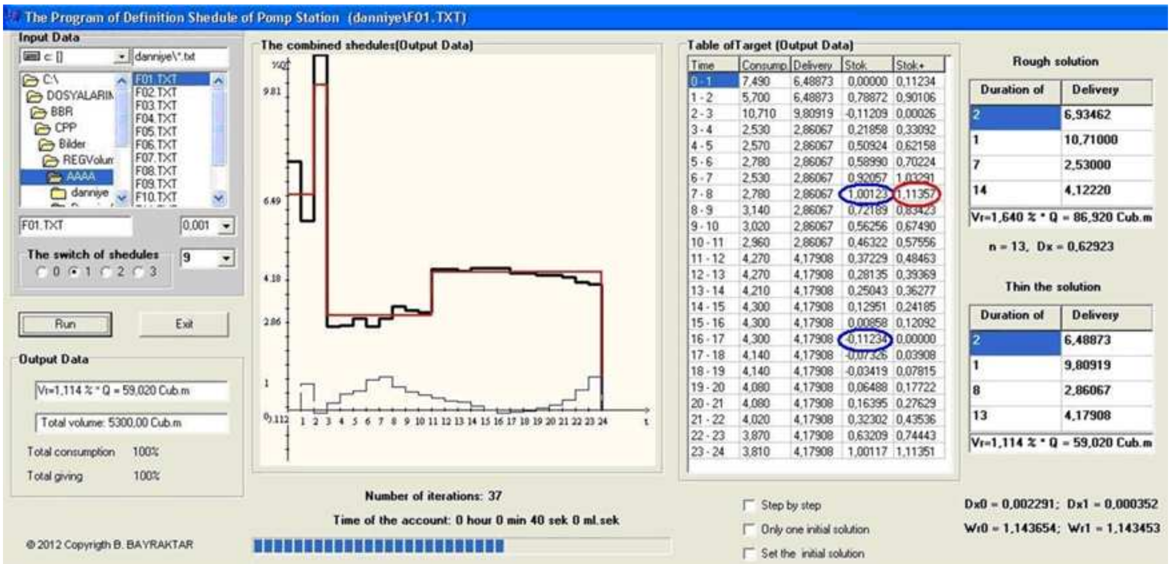

Figure 5. Screen from the program.

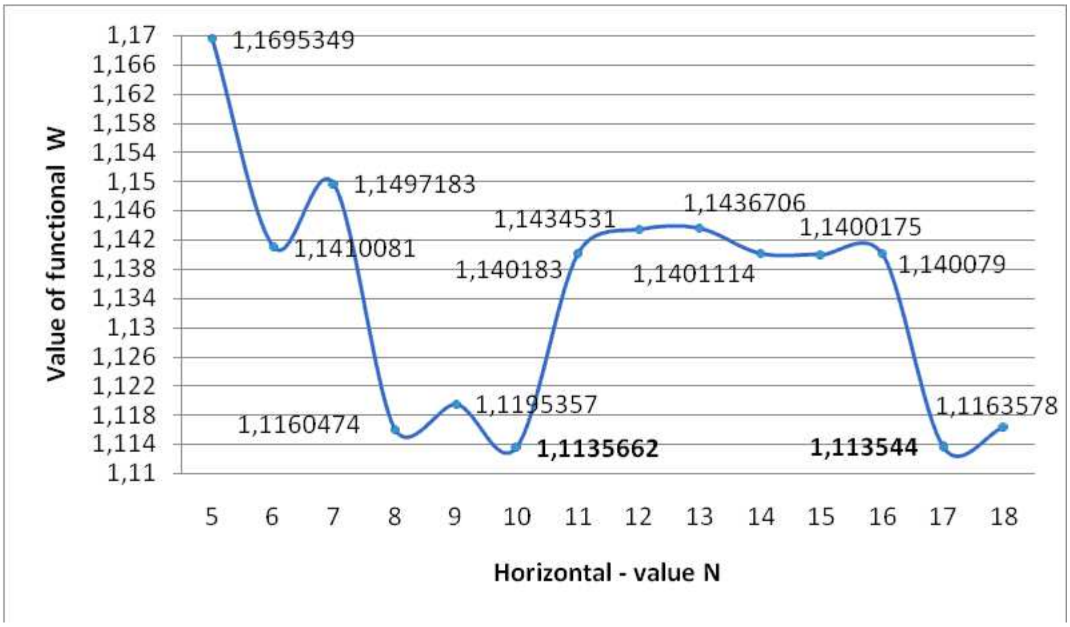

Figure 6. Best solutions constructed based on 14 initial solutions.

7. Some discussions

It is evident that the finer the decomposition of an interval is, the better solution is; but the numerical experiments showed that the problem solution does not change practically at a significant increase in number of initial solutions and number of parts of the search interval decomposition, respectively.

The Table 1 below represents the summarized data of iteration processes ($6 \leq N \leq 35, N = 50$ and $N = 100$). For the illustrative purposes, the same data are represented in the form of a plot in Fig. 9. In the table the values of N (interval decomposition), at which the best solutions of approximately one order are obtained, are in bold is selected in Input Data bar. If it is assumed that the value $W = 1.113485$ obtained at $N = 28$ is the optimal value of the functional then it

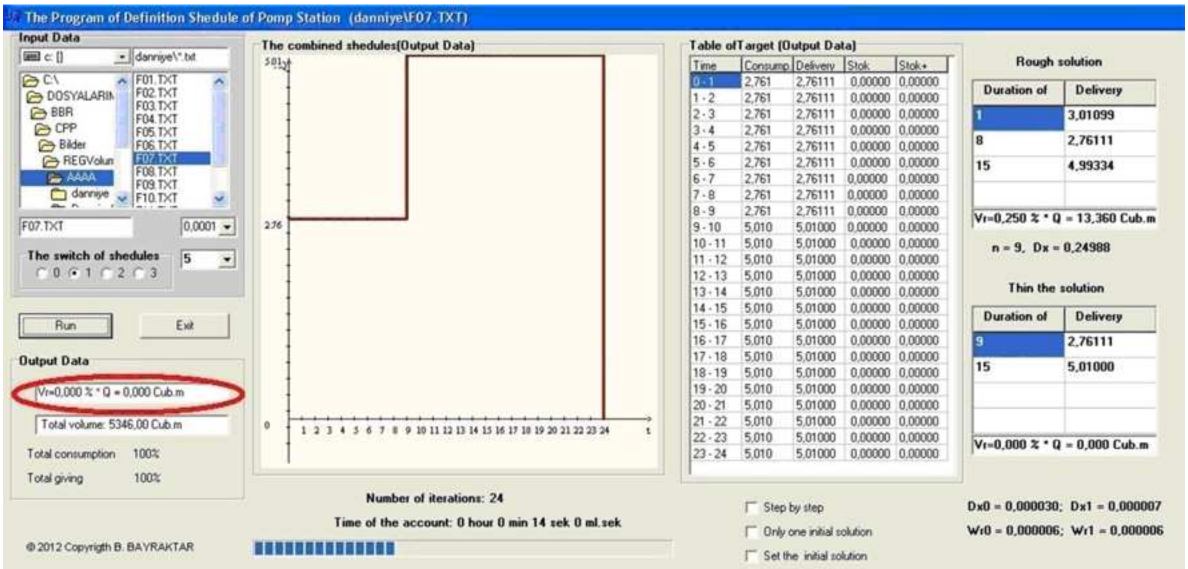

Figure 7. For 2-step plot of consumption the absolute optimum is found to be W=0.000 in course of 24 iterations. The number of given initial solutions 5, time of calculation is 14 sec.

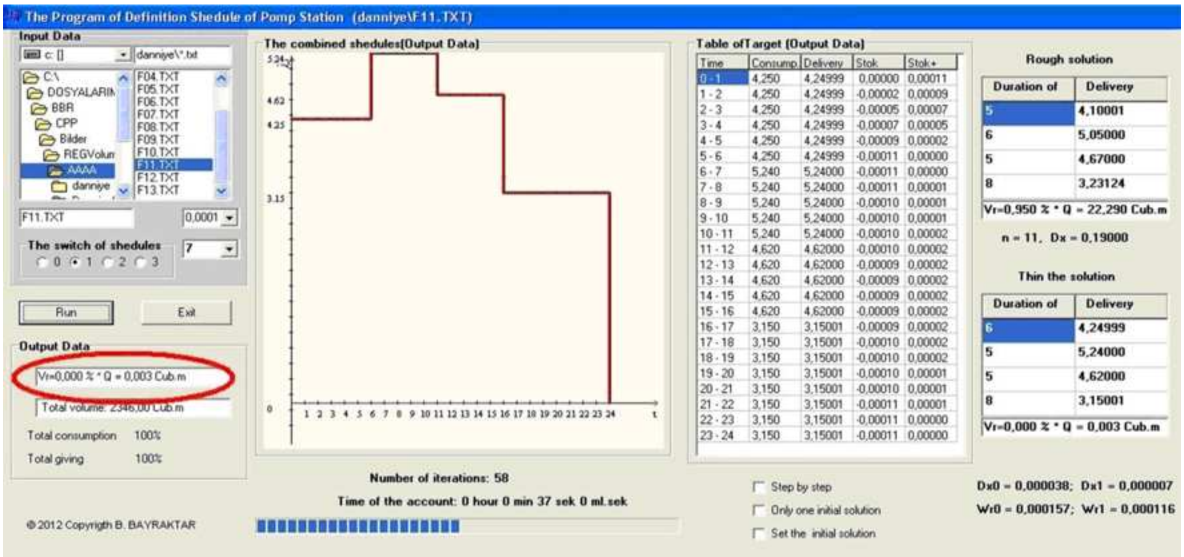

Figure 8. For 4-step plot of consumption the absolute optimum is found to be W=0.000 in course of 58 iterations. The number of given initial solutions 7, time of problem solution 37 sec.

is seen from the table that the problem solution of such order was obtained at less N ($N = 10$, 17, 19 etc.).

All this is observed also for each of nine test cases. In my opinion, this fact demonstrates the efficiency of the suggested algorithm since the possibility to decrease the N value means a multiple reduction of the problem time.

Table 1.

Interval partition N	**Rough solution (W_0)**	**Thin solution (W)**	**Interval partition N**	**Rough solution (W_0)**	**Thin solution (W)**
6	1.77700	1.141008	22	1.26909	1.140800
7	1.64003	1.149718	23	1.18561	1.140013
8	1.64003	1.116047	24	1.19796	1.115170
9	1.64003	1.119536	**25**	**1.20376**	**1.113587**
10	**1.64003**	**1.113566**	26	1.21183	1.130306
11	1.64003	1.140183	**27**	**1.21932**	**1.113523**
12	1.64003	1.143453	**28**	**1.22629**	**1.113485**
13	1.64003	1.143671	29	1.23280	1.142616
14	1.64003	1.140111	30	1.23889	1.117425
15	1.64003	1.140018	31	1.24459	1.154496
16	1.64003	1.140079	32	1.24914	1.114602
17	**1.73219**	**1.113544**	**33**	**1.19967**	**1.113500**
18	1.61260	1.116358	**34**	**1.17261**	**1.113517**
19	**1. 50497**	**1.113524**	**35**	**1.15844**	**1.113539**
20	**1.40759**	**1.113651**	50	1.11954	1.119539
21	1.31906	1.140008	**100**	**1.11954**	**1.113490**

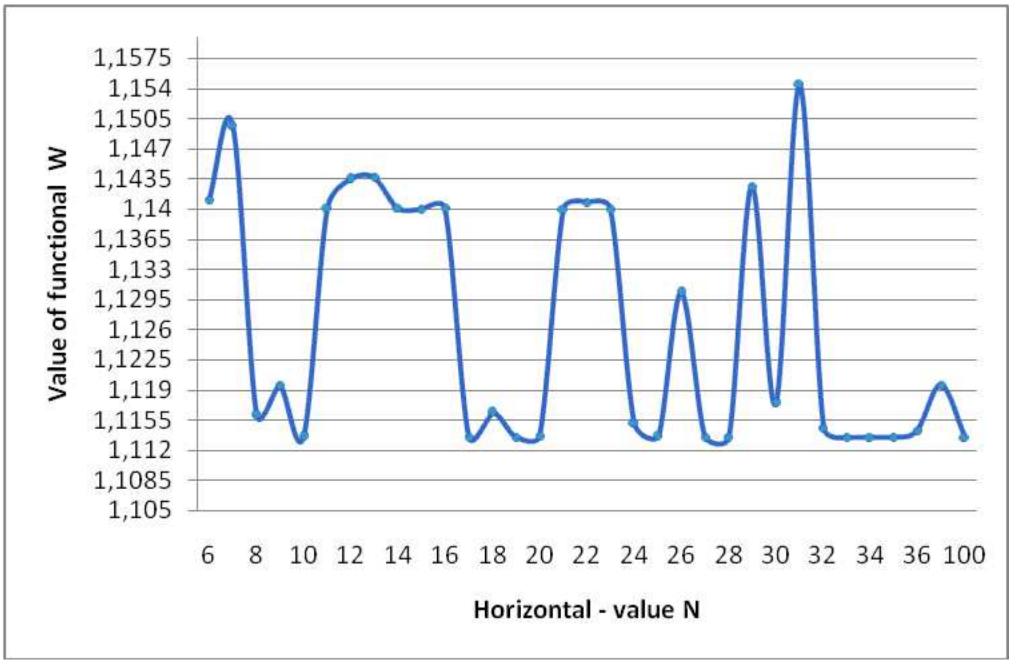

Figure 9. Graphical presentation of Table 1 data.

Table 2 shows the calculation results and time of calculations for various objects. The same objects were calculated using the algorithm of directed exhaustion with step being equal to the accuracy of the basic data of water consumption (Δ=0.01); the solutions of the problem of the same order (not better) were obtained; however, the time for calculation took dozens of hours.

Table 2.

Algorithm of narrowing neighborhoods (solution accuracy $\varepsilon = 0.001$)					
The object	**W**	**Counting time**	**The object**	**W**	**Counting time**
F01.txt	1,1140	00m, 41sec	**F05.txt**	0,0000	00m, 14sec
F02.txt	2,0710	00m, 24sec	**F06.txt**	0,0000	00m, 42sec
F03.txt	1,8220	00m, 28sec	**F07.txt**	0,0000	00m, 14sec
F04.txt	0,3370	00m, 42sec	**F08.txt**	0,0030	00m, 22sec
F09.txt	1,0670	00m, 10sec	**F11.txt**	0,0000	00m, 37sec

The software of algorithm using up – to – date tools of visual programming (Builder 6.0 C++) was developed by me. The numerical experiments were conducted based on computer HP Compaq 6000 Pro MT PC Intel(R) Corel TM) Duo CPU E7500 @ 2.93Ghz, 1.58 GHz.

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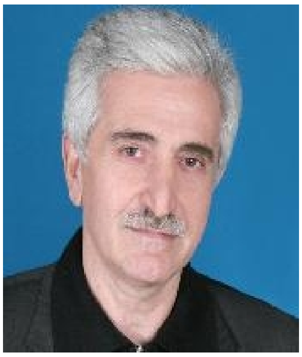

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